Introduction

Kharkiv State University of Food Technology and Trade (Kharkiv, Ukraine) has been carrying out investigations aimed at the creation of long-term nutrition systems for the prevention and treatment of the diseases caused by calcium deficiency for many years.

There are two factors specifying the actuality of the defined problem. First of all, about 75% of all diseases are directly or indirectly connected with the unhealthy diet [1]. Secondly, many diseases cause the necessity of providing a sick person with balanced food for a long time (for weeks, months, years).
Scientifically substantiated balanced nutrition means that a diverse but always large totality of correlations between different nutrients (the amount of fat and calcium, calcium and phosphorus, calcium and magnesium), between the contents of amino acids, between the groups of fatty acids, etc. are to be observed for various categories of consumers. Daily needs in all essential nutrients are to be satisfied. Besides, the contents of the ingredients in the diets are to be physiologically grounded and convenient for processing. It means that technological conditions and restrictions are to be fulfilled.

It follows from the above that it is impossible to create the system of balanced nutrition without the science about quantitative ratio – mathematics, without a qualified and creative use of different mathematical toolkit and modern software.

Literature review

At present there are a lot of publications devoted to the projection of the recipes of specific products and dishes [2-7]. At the same time there are very few works devoted to the investigation of the problems concerning the projection of a meal or expendable diets (ED): for breakfast, dinner, supper, etc [8-9]. There are even fewer works dedicated to the projection of daily diets (DD), and very small quantity of the books cover the questions concerning nutrition systems (NS) design

In the work [10] the main principles for the formation of nutrition systems were formulated:

a) nutrition systems are based on the use of mass consumption ingredients;

b) mathematical models, methods and modern software are to be used at all stages of nutrition systems formation and analysis of their efficiency, etc.

The authors specified the structure of the nutrition systems based on:

a) the complexes of expendable diets (ED) of various types (breakfast and lunch, dinner and supper);

b) the complexes of expendable diets (ED) consisting of three-four EDs of different purposes;

c) cyclic diets.

In their works [11-12] the authors presented the methodology of mathematical modelling and optimization of the ingredients’ contents in the complexes of expendable diets (ED) for various intended purposes. The article [13] is devoted to the creation of mathematical models for daily diets optimization.

At the previous stages of the research the authors developed a large totality of mathematical models of expendable diets (ED) for various intended purposes, and optimized them.

The projects of three types of nutrition systems are created. In an average they ensure a daily, day-and-a-half and 2-days level of needs in a balanced calcium.

The aim of the research is:

1. To analyze mathematical aspects for the creation of durable nutrition systems aimed at the prevention and medication of the diseases caused by calcium deficiency.

2. Specify the ways for the improvement of the developed mathematical toolkit with the aim of projecting systems with a higher level of both balancing the nutrients, and meeting daily requirements in them.

Results and discussion

In the process of projecting nutrition systems the authors created and used the following mathematical toolkit:

1. The totality of mathematical models of expendable diets (ED) for various intended purposes and methods for the optimization of the ingredients content in them.

2. Mathematical optimization models for daily diets (DD), which are based on the ED totality.

3. The formalized method of fast and light determination of ED biological value; the method based on linear dependencies of scores of essential amino acids in a diet on the similar scores of the ingredients.

Mathematical models of ED contain:

a) main physiological restrictions and correlations between the amount of fat and calcium in the diets, calcium and phosphorus, calcium and magnesium;

b) technological restrictions for the amount of the ingredients in a diet;

c) the conditions for the enrichment of the diet with nutrients influencing metabolic processes of bone tissue;

d) energetic value conditions;

f) various objective functions for different diets.

The mentioned mathematical models of the expendable diets (ED) are the models of linear programming problems. The ingredients content in EDs was optimized by means of a simplex method.

Mathematical models of daily diets optimization are the models of the integral mathematical programming with Boolean variables.

The method developed by the authors and used for calculation of the scores of essential amino acids is the method of linear algebra.

During the analysis of three basic nutrition systems (NS) an average daily amount of nutrients for each system was determined. Analysis of the nutrition systems projected by means of the suggested mathematical toolkit demonstrated that:
1. The suggested nutrition systems on an average provide daily requirements in a large amount of important nutrients;

2. Nutrition systems of the first generation procure the fulfillment of the recommended by nutrition physiologists correlations between the content of fats and calcium, calcium and phosphorus, calcium and magnesium, both on the level of expendable and daily diets;

3. At the same time, out of 23 nutrients influencing metabolic processes in the bone tissue, the content of which in food ingredients used in Ukraine is well known, the most deficient nutrients in food systems of all three kinds are: selenium, fluorine and barium, less deficient are zinc, manganese and iodine. In nutrition systems of the first type, vitamins B6, B2, E, silicon and proteins are added to the totality of the deficient nutrients besides the mentioned ones. It is worth saying that some deficiency of carbohydrates by the side of the norms used for consumption in Ukraine is observed in the nutrition systems of the first and second types. It is caused by voluntary reduction of the amount of the ingredients, which are the sources of the easily digested carbohydrates used in nutrition systems. The reasons for this are the following. First, it happens because in the modern world the diets of many individuals are characterized by the excess of carbohydrates [14]. Secondly, there is some scientific information concerning the provocative influence of carbohydrates excess in the diets on oncological diseases [15]. Secondly, taking into account all the above mentioned, the carbohydrates are not considered as a deficient nutrient in the created nutrition systems.

4. There is also a chain of correlations between the nutrients recommended by the nutritionists, or groups of nutrients, which have not yet been accounted in mathematical models, but which are to be used for the creation of nutrition systems of the coming generation. The following belong to such correlations:
   a) amid proteins, fats and carbohydrates;
   b) amid ten essential amino acids;
   c) amid the contents of fatty acids (saturated, monounsaturated and polyunsaturated).

   So, the presented results prove the reasonability of further increase both of the nutrients balancing and provision of daily needs in them. It means that the problem of creating effective nutrition systems remains topical, and the search of the methods for the improvement of mathematical toolkit alongside with the nutrition systems projected with its help, should be continued.

At the stage of projecting nutrition systems of the second generation, i.e. more consummate NS, let’s implement the following three approaches:

1. To provide diets and nutrition systems in general with the deficient nutrients or at least reduce scarcity in many nutrients let’s project (with the use of more complicated mathematical toolkit) the recipes of floury products enriched with the deficient nutrients. They may further be used in mathematical models of expendable diets.

2. With the aim of increasing the level of nutrients balancing both at the stage of products projecting, and at the stage of creating EDs, let’s additionally take into account entangled relations between the nutrients from each group besides the limitations on correlations between the nutrients already taken into consideration. These are the relations amid:
   a) the proteins, fats and carbohydrates;
   b) ten essential amino acids;
   c) the content of fatty acids (saturated, monounsaturated and polyunsaturated).

3. Target functions will be improved in the models of the ingredients content optimization for EDs.

   Let’s review the approaches in detail.

   During the mathematical modeling of both recipes for the dishes and expendable diets, the following mathematical problem may arise. In some cases, when the total amount of restrictions (primarily, the ones providing essential correlations between the pairs of different nutrients) is extremely large, the tasks of linear programming do not have solutions.

   To take the influence of various groups of the related scientifically substantiated correlations of nutrients into consideration in the models, and simultaneously ensure solutions of the problems of linear programming (both for the products and for the diets), the authors introduced the functionals of balancing the related groups of nutrients: the functional of proteins, fats and carbohydrates, the functional of ten essential amino acids, and the functional of a group of fatty acids (saturated, monounsaturated, polyunsaturated), and aggregated limitations on the functional magnitude.

   The weighed sum of the contents of the group’s nutrients (g) is called the functional of balancing of the nutrients’ groups related by scientifically substantiated physiological correlations. Numerically it equals the sum of the products of the nutrients’ contents (g) (according to the product’s recipe, ED or DD) multiplied by the coefficients of the nutrients ponderability. It is a mathematical expression of the development of a group of nutrients, which are related by the recommended physiological correlations.

   We proposed to introduce additionally limitations on the weighed sums of the related nutrients into mathematical models but not on the correlations between the pairs of nutrients.

   Introduction of the functionals of balancing nutrients and aggregated limitations to their volumes will allow, first, significantly improve mathematical models for projecting some products, EDs and DDs, taking into consideration large groups of nutrients without any notable complication of the models, i.e. without any considerable increase of the total amount of limitations. Secondly, it will be possible to receive the solutions of the corresponding linear programming tasks, and, as a conclusion, to create perspectives for projecting products, EDs, DDs and NSs in general with a higher level of nutrients balancing.

   Hereinafter, to illustrate the abovementioned, the authors present the mathematical model of projecting expendable diet of the second generation, in which a separately projected floury product enriched with deficient...
nutrient selenium is used. It is a selenic roll. Aggregated limitations on functionals of balancing three important groups
of nutrients are introduced in the model.

Maximum one of three functionals of nutrients balancing – maximum functional of balancing proteins, fats and
carbohydrates is chosen as a target function.

It is worth mentioning that the recipe of a selenic roll is projected with the use of mathematical model, analogous
to the one presented beneath. This product (100 g) covers daily requirements of the body in organic selenium. Also it
is enriched with a lot of deficient nutrients.

<table>
<thead>
<tr>
<th>Mathematical model of ED for dinner</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salad, solyanka with mushrooms, veal liver with garnish, dessert</td>
</tr>
</tbody>
</table>

**I. Technological limitations on the ingredients contents in a diet**

**Main ingredients of the salad:**
- Leaf lettuce: $30 \leq X_1 \leq 40$ (1)
- Dutch cheese bar: $30 \leq X_2 \leq 55$ (2)
- The core of hazel-nut: $10 \leq X_3 \leq 15$ (3)

**The first dish:**
- Solyanka with mushrooms (recipe №176): $X_4 = 250$ (4)

**Main ingredients of the second dish:**
- Veal liver: $80 \leq X_5 \leq 120$ (5)
- Lens: $15 \leq X_6 \leq 35$ (6)
- Green head cabbage: $50 \leq X_7 \leq 100$ (7)

**Fruits**
- Persimmon: $80 \leq X_9 \leq 120$ (8)

**Baked goods:**
- Selenic roll: $50 \leq X_{10} \leq 120$ (9)
- Wheat bread: $0 \leq X_{11} \leq 60$ (10)

**Additional ingredients:**
- Salt: $1 \leq X_{12} \leq 2$ (11)
- Sun seed oil: $5 \leq X_{13} \leq 20$ (12)
- Sour cream, 20% fat: $20 \leq X_{14} \leq 30$ (13)
- Mineral water Borzhomy: $100 \leq X_{15} \leq 200$, (14)

where $X_i, i=1, 2, \ldots 14$ is x-amount of the ingredient (g) of $i$-type in a projected diet, $X_i \geq 0$.

The correlations connecting the contents of nutrients $Y_j$ with the ingredients are:

$$ Y_j = \sum_{i=1}^{14} a_{ij} \cdot X_i, $$

where $a_{ij}$ is the amount of $j$-type nutrient in 1g of $i$ ingredient.

**II. Main physiological limitations on correlations:**

- between the amount of fat and calcium: $66 \leq \frac{Y_1}{Y_2} \leq 85$; (16)
- between the amount of fat, calcium and phosphorus: $0.90 \leq \frac{Y_1}{Y_3} \leq 1.1$; (17)
- between the amount of calcium and magnesium: $2.9 \leq \frac{Y_2}{Y_4} \leq 3.7$; (18)

$Y_1, Y_2, Y_3, Y_4$ are the contents of calcium, fat, phosphorus and magnesium in a projected diet.

**III. Aggregated restrictions on functionals of balancing groups of nutrients (conditions for the enrichment
ED with the balanced groups of nutrients)**

1. Limitations on functional of balancing proteins, fats and carbohydrates

$$ \frac{\Phi_{pfc}^{ED}}{\Phi_{pfc}^{nc}} \cdot 10^2 \geq 30\% , $$

(19)
where

\[ \Phi_{pfc}^{ED} = \frac{1}{7} Y_5 + \frac{1}{7} Y_2 + \frac{5}{7} Y_6, \]  

(20)

\( \Phi_{pfc}^{ED}, \Phi_{pfc}^{dn} \) – are the functionals of balancing proteins, fats and carbohydrates according to ED and daily requirements;  
\( Y_5, Y_6 \) are respectively the contents (g) of proteins, fats and carbohydrates in a projected diet.

2. Limitations on the functional of balancing fatty acids:

\[ \frac{\Phi_{fa}^{ED}}{\Phi_{fa}^{dn}} \cdot 10^2 \geq 50\%, \]  

(21)

where

\[ \Phi_{fa}^{ED} = \frac{1}{3} Y_{sfa} + \frac{1}{3} Y_{mufa} + \frac{1}{3} Y_{pufa}, \]  

(22)

\( Y_{sfa}, Y_{mufa}, Y_{pufa} \) show the contents of saturated, monounsaturated and polyunsaturated fatty acids.

3. Limitations on the functional of balancing essential amino acids:

\[ \frac{\Phi_{ea}^{ED}}{\Phi_{ea}^{dn}} \cdot 10^2 \geq 50\%, \]  

(23)

where

\[ \Phi_{ea}^{dn} = \sum_{i=36}^{45} \alpha_i Y_i, \]  

(24)

\[ \alpha_{36} = \frac{4}{33,75}, \alpha_{37} = \frac{3.5}{33,75}, \alpha_{38} = \frac{5}{33,75}, \alpha_{39} = \frac{4}{33,75}, \alpha_{40} = \frac{3}{33,75}, \]  

\[ \alpha_{41} = \frac{2.5}{33,75}, \alpha_{42} = \frac{1}{33,75}, \alpha_{43} = \frac{3}{33,75}, \alpha_{44} = \frac{6}{33,75}, \alpha_{45} = \frac{1.75}{33,75}; \]  

(25)

where \( Y_{36}, Y_{37}, \ldots, Y_{45} \) demonstrate the amount (g) of essential amino acids respectively: valine, isoleucine, leucine, lysine, methionine, threonine, tryptophan, phenylalanine, arginine, histidine in the projected diet;  
\( \Phi_{ea}^{ED}, \Phi_{ea}^{dn} \) reflect the volumes of the functionals of balancing essential amino acids corresponding to their amount in an ED and daily requirements in them.

IV. The conditions for the enrichment of the diet with deficient nutrients:

- selenium \[ \frac{Y_{19}}{Y_{19}} \cdot 10^2 \geq 90\%; \]  

(26)

- boron \[ \frac{Y_{17}}{Y_{17}} \cdot 10^2 \geq 80\%; \]  

(27)

- fluorine \[ \frac{Y_{20}}{Y_{20}} \cdot 10^2 \geq 95\%; \]  

(28)

- manganese \[ \frac{Y_{18}}{Y_{18}} \cdot 10^2 \geq 40\%; \]  

(29)

- iodine \[ \frac{Y_{11}}{Y_{11}} \cdot 10^2 \geq 60\% \]  

(30)

- zinc \[ \frac{Y_{10}}{Y_{10}} \cdot 10^2 \geq 70\%. \]  

(31)

V. Умови збагачення раціону менш дефіцитними нутрієнти:

- calcium \[ \frac{Y_{1}}{Y_{1}} \cdot 10^2 \geq 70\%; \]  

(32)

- fat \[ \frac{Y_{2}}{Y_{2}} \cdot 10^2 \geq 50\%; \]  

(33)

- phosphorus \[ \frac{Y_{3}}{Y_{3}} \cdot 10^2 \geq 50\%; \]  

(34)
– magnesium \( \frac{Y_4}{Y_{40}} \cdot 10^2 \geq 50\% \); \( (35) \)
– proteins \( \frac{Y_5}{Y_{50}} \cdot 10^2 \geq 70\% \); \( (36) \)
– carbohydrates \( \frac{Y_6}{Y_{60}} \cdot 10^2 \geq 25\% \); \( (37) \)
– iron \( \frac{Y_8}{Y_{80}} \cdot 10^2 \geq 60\% \); \( (38) \)
– vitamin D \( \frac{Y_9}{Y_{90}} \cdot 10^2 \geq 25\% \); \( (39) \)
– potassium \( \frac{Y_{12}}{Y_{120}} \cdot 10^2 \geq 50\% \); \( (40) \)
– vitamin C \( \frac{Y_{13}}{Y_{130}} \cdot 10^2 \geq 90\% \); \( (41) \)
– vitamin B_2 \( \frac{Y_{14}}{Y_{140}} \cdot 10^2 \geq 80\% \); \( (42) \)
– vitamin B_6 \( \frac{Y_{15}}{Y_{150}} \cdot 10^2 \geq 55\% \); \( (43) \)
– copper \( \frac{Y_{16}}{Y_{160}} \cdot 10^2 \geq 50\% \); \( (44) \)
– silicone \( \frac{Y_{21}}{Y_{210}} \cdot 10^2 \geq 60\% \); \( (45) \)
– vitamin A \( \frac{Y_{22}}{Y_{220}} \cdot 10^2 \geq 65\% \); \( (46) \)
– vitamin E \( \frac{Y_{23}}{Y_{230}} \cdot 10^2 \geq 70\% \); \( (47) \)
– sodium \( \frac{Y_{24}}{Y_{240}} \cdot 10^2 \geq 40\% \). \( (48) \)

VI. The condition for energy value
\[ \frac{Y_7}{Y_{70}} \cdot 10^2 \geq 40\%. \] \( (49) \)

VII. Target function: maximum of the functional of balancing proteins, fats and carbohydrates
\[ Z = \Phi_{pk} = \frac{1}{7} Y_5 + \frac{1}{7} Y_2 + \frac{5}{7} Y_6 \rightarrow \max. \] \( (50) \)

Mathematical formulation of the problem on the optimization of the contents of the ingredients in a diet is to specify the vector \( \vec{X} = (x_1, x_2, \ldots, x_{14}) \), which maximizes the target function (50) upon condition that this vector’s coordinates satisfy the system of equations and inequalities (1) – (49). Results of the presented diet are demonstrated in Table 1.

<table>
<thead>
<tr>
<th>Ingredient</th>
<th>Optimal content in a diet, g</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Leaf lettuce</td>
<td>40</td>
<td>Persimmon</td>
<td>120</td>
</tr>
<tr>
<td>Dutch cheese bar</td>
<td>55</td>
<td>Selenic roll</td>
<td>120</td>
</tr>
<tr>
<td>The core of hazel-nut</td>
<td>10</td>
<td>Wheat bread</td>
<td>60</td>
</tr>
<tr>
<td>Solyanka with mushrooms (recipe № 176)</td>
<td>250</td>
<td>Salt</td>
<td>2</td>
</tr>
<tr>
<td>Veal liver</td>
<td>80</td>
<td>Sun seed oil</td>
<td>20</td>
</tr>
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<tr>
<td>Green head cabbage</td>
<td>100</td>
<td>Mineral water Borzhomy</td>
<td>200</td>
</tr>
</tbody>
</table>
Based on the analysis of the data presented in Table 1, it is found that the obtained optimal values of the ingredients content in the projected diet correspond the previously determined technological restrictions (1) – (14).

It should be noted that due to such content of the ingredients in the diet it is possible to achieve optimum correlation between the nutrients influencing calcium absorption – 67,38 for the correlation fat:calcium; 0,95 for the correlation calcium:phosphorus, and 3,39 for the correlation calcium:magnesium that corresponds to the established physiological restrictions (16) – (18). The projected diet also allows satisfy daily needs in nutrients and energy on a high level that was provided by the restrictions (29) – (49) and the selection of the appropriate ingredients with a valuable nutritive composition.

Conclusion

The analysis of mathematical toolkit created for the projection of three basic healthful and dietary nutrition systems, which cover daily, day-and-a-half and two-day requirements in balanced calcium for the period multiple of a fortnight, is performed.

The ways for the improvement of mathematical toolkit for the nutrition systems projecting are determined. They are:

a) the creation of functionals of balancing the groups of nutrients connected by scientifically substantiated correlations, and their introduction at all stages of nutrition systems projecting – designing the recipes of dishes, expendable and daily diets;

b) the projection (by means of mathematical models and methods) of the totality of the recipes of floury products enriched with deficient nutrients and their further use in EDs;

c) the improvement of mathematical models for the optimization of the ingredients amount in multipurpose expendable diets, among which is the use of aggregated limitations on functionals of balancing the groups of nutrients;

d) the creation of the second generation of nutrition systems in general, the systems with a higher level of both nutrients balance and provision of daily requirements in them.

The suggested approaches can be used for both nutrition systems aimed at the prevention and medication of the diseases caused by calcium deficiency, and durable diverse nutrition systems are created.

References:
Аннотация. Проанализирован математический инструментарий, который разработан и использован для проектирования долговременных систем питания, предназначенных для профилактики и лечения заболеваний, возникающих на фоне дефицита кальция, в частности совокупности математических моделей разового потребления и методы оптимизации содержания ингредиентов в них, математические модели оптимизации суточных рационов, а также формализованный метод быстрого и лёгкого определения биологической ценности рационов.

Определены пути совершенствования разработанного математического инструментария с целью создания систем питания с более высоким уровнем как сбалансирования нутриентов, так и обеспечения суточных потребностей в них на основе использования в составе рационов питания нетрадиционных мучных изделий, обогащённых дефицитными нутриентами, функционал сбалансирования связанных между собой групп нутриентов, а также введения в модели (как изделия, так и рационы) агрегированных ограничений на эти группы нутриентов.

Ключевые слова: системы питания, математические модели, целевые функции, задачи линейного программирования, функционалы сбалансирования групп нутриентов, агрегированные ограничения, задачи целочислового программирования с булевыми переменными.

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